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A SIMPLE PHYSICAL APPROACH FOR DERIVING THE CHARACTERISTIC EQUATIONS OF FLUID DYNAMICS

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SUMMARY

A simple physical approach for deriving the characteristic equations of fluid dynamics is presented. The approach is based on the physical concept that information propagates through a flowfield along pathlines due to particle motion and along wavelines due to acoustic wave motion. The characteristic equations and compatibility equations are derived in vector forms which are valid in any co-ordinate system.

KEY WORDS Characteristics Compressible flow

INTRODUCTION

The method of characteristics is one of the most important methods for solving the governing equations of fluid dynamics. It shows that there are special surfaces in the solution space on which linear combinations of the governing partial differential equations can be formed that contain derivatives only in the special surfaces themselves. These special surfaces are called characteristic surfaces, and the linear combinations of the governing partial differential equations that apply on these surfaces are called compatibility equations.

Two families of characteristic surfaces exist. One family of characteristic surfaces consists of stream surfaces, which are surfaces composed of pathlines. The envelope of all of the stream surfaces passing through a point in the flowfield is the unique pathline passing through the point. The other family of characteristic surfaces consists of wave surfaces, which are surfaces composed of acoustic waves. The envelope of all wave surfaces at a point in the flowfield is the Mach conoid. The line of contact between a particular wave surface and the Mach conoid is called a waveline (bicharacteristic and ray path are more commonly used). The Mach conoid is composed of an infinite number of wavelines.

The compatibility equations that are valid along the stream surfaces and wave surfaces can be written as interior operators within those surfaces, which have derivatives only within those surfaces. In particular, they can be written as directional derivatives along the pathline and the wavelines passing through each point in the flowfield. Information is propagated through the flowfield along these paths. The method of characteristics yields critical insights into the range of influence of initial data and the domain of dependence of solution points. The method of characteristics also gives insights into the proper specification of boundary conditions.

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There are several ways to derive the characteristic and compatibility equations from the governing partial differential equations.[1–6] For unsteady one-dimensional flow or steady two-dimensional supersonic flow, the mathematical procedures are straightforward. However, for unsteady two- and three-dimensional flows, the mathematical complexity of the existing derivations (e.g. Rusanov[3] and Hoffman[6]) conceals the physical concepts that underly the method of characteristics. Apparently this has caused most computational fluid dynamicists to ignore this powerful approach to understanding and solving unsteady two- and three-dimensional flow problems.

The objective of this work[7] was to derive the characteristic and compatibility equations by a simple physical approach. The derivation is based on the physical concept that information propagates through a flowfield along pathlines due to particle motion and along wavelines due to acoustic wave motion. The characteristic and compatibility equations are presented in vector forms which are valid in any co-ordinate system.

GOVERNING EQUATIONS

The governing equations for unsteady one-, two- or three-dimensional flow are the continuity equation, the momentum equation and the energy equation. Those equations are[4]

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} + \rho\nabla\cdot\mathbf{V} = 0, \tag{1}$$

$$\rho\frac{\mathrm{D}\mathbf{V}}{\mathrm{D}t} + \nabla P = \mathbf{F}, \tag{2}$$

$$\frac{\mathrm{D}P}{\mathrm{D}t} - a^2\frac{\mathrm{D}\rho}{\mathrm{D}t} = \psi, \tag{3}$$

where $\mathrm{D}\rho/\mathrm{D}t$, $\mathrm{D}\mathbf{V}/\mathrm{D}t$ and $\mathrm{D}P/\mathrm{D}t$ are particle derivatives, commonly called substantial or material derivatives, which are directional derivatives along the pathline, and $\mathbf{F}$ and ψ are source terms, if present. The operator $\mathrm{D}(\cdot)/\mathrm{D}t$ is defined as

$$\frac{\mathrm{D}(\cdot)}{\mathrm{D}t} = \frac{\partial(\cdot)}{\partial t} + \mathbf{V}\cdot\nabla(\cdot). \tag{4}$$

Equations (1) and (3) can be combined to eliminate the derivatives of density. Thus

$$\frac{\mathrm{D}P}{\mathrm{D}t} + \rho a^2\nabla\cdot\mathbf{V} = \psi. \tag{5}$$

The thermal equation of state and the speed of sound are specified by the functional relationships

$$T = T(P, \rho) \quad \text{and} \quad a = a(P, \rho). \tag{6}$$

PARTICLE MOTION AND ACOUSTIC WAVE MOTION

Information propagates through a flowfield by particle motion and acoustic wave motion as shown in Figure 1. Figure 1 illustrates a change in the position vector $\mathbf{r}$ by $\mathrm{d}\mathbf{r}$. When $\mathrm{d}\mathbf{r}$ is aligned in the direction of the particle velocity $\mathbf{V}$,

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \mathbf{V} = \frac{\mathrm{D}\mathbf{r}}{\mathrm{D}t} \quad \text{(along pathlines)}, \tag{7}$$

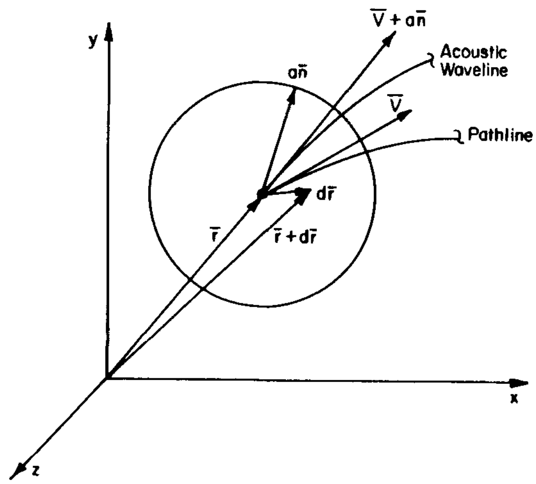

Figure 1. Particle motion and acoustic wave motion

where dt is the time required for the fluid particle to move the displacement **dr**. In Cartesian co-ordinates, equation (7) is

$$\mathrm{D}x = u\,\mathrm{D}t, \qquad \mathrm{D}y = v\,\mathrm{D}t, \qquad \mathrm{D}z = w\,\mathrm{D}t, \tag{8}$$

where D(·) denotes the differential along the pathline and u, v and w are the particle velocity components in the x-, y- and z-directions respectively.

For acoustic wave motion, information propagates in all directions from a point at the speed of sound a relative to the particle velocity **V**. Thus the absolute velocity of wave motion $\mathbf{V}_w$ is the sum of the particle velocity **V** and the relative velocity of the acoustic wave in the direction of the arbitrary unit vector **n**, which is $a\mathbf{n}$. Thus

$$\frac{\mathbf{dr}}{\mathrm{d}t} = \mathbf{V}_w = \mathbf{V} + a\mathbf{n} = \frac{\mathscr{D}\mathbf{r}}{\mathscr{D}t} \quad \text{(along wavelines)}, \tag{9}$$

where dt is the time required for the acoustic wave to move the displacement **dr**. In Cartesian co-ordinates, equation (9) is

$$\mathscr{D}x = (u + an_x)\mathscr{D}t, \qquad \mathscr{D}y = (v + an_y)\mathscr{D}t, \qquad \mathscr{D}z = (w + an_z)\mathscr{D}t, \tag{10}$$

where $\mathscr{D}(\cdot)$ denotes the differential along the waveline and n_x, n_y and n_z are the components of **n** in the x-, y- and z-directions respectively. The absolute velocity of propagation of acoustic waves relative to the co-ordinate axes depends on the particle velocity **V**, the speed of sound a and the direction **n**. Thus there are an infinite number of acoustic wave velocities corresponding to the infinite number of choices for the direction of the arbitrary unit vector **n**, even though the speed of sound relative to the particle has only one value (i.e. a) at a point in the flowfield at a given time.

PARTICLE DERIVATIVE AND WAVE DERIVATIVE

According to the Eulerian description of fluid flow, any fluid property f is expressed as a function of the time t and the space co-ordinate **r**. Thus

$$f = f(t, \mathbf{r}). \tag{11}$$

The total derivative of f with respect to time t is given by

$$\frac{\mathrm{d}f}{\mathrm{d}t} = f_t + \nabla f \cdot \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}, \tag{12}$$

which in Cartesian co-ordinates is

$$\frac{\mathrm{d}f}{\mathrm{d}t} = f_t + f_x \frac{\mathrm{d}x}{\mathrm{d}t} + f_y \frac{\mathrm{d}y}{\mathrm{d}t} + f_z \frac{\mathrm{d}z}{\mathrm{d}t}. \tag{13}$$

The value of $\mathrm{d}\mathbf{r}$ in equation (12) is arbitrary, so that equation (12) specifies $\mathrm{d}f$ for any arbitrary change of position $\mathrm{d}\mathbf{r}$.

Two special choices for $\mathrm{d}\mathbf{r}/\mathrm{d}t$ are of interest: $\mathrm{d}\mathbf{r}/\mathrm{d}t$ for particle motion and $\mathrm{d}\mathbf{r}/\mathrm{d}t$ for acoustic wave motion. Substituting equation (7) for particle motion into equation (12) yields

$$\left.\frac{\mathrm{d}f}{\mathrm{d}t}\right|_{\text{pathline}} = f_t + \mathbf{V} \cdot \nabla f = \frac{\mathrm{D}f}{\mathrm{D}t}, \tag{14}$$

where $\mathrm{d}f/\mathrm{d}t$ denotes the directional derivative following a fluid particle, which is the substantial derivative $\mathrm{D}f/\mathrm{D}t$. Substituting equation (9) for acoustic wave motion into equation (12) yields

$$\left.\frac{\mathrm{d}f}{\mathrm{d}t}\right|_{\text{waveline}} = f_t + (\mathbf{V} + a\mathbf{n}) \cdot \nabla f = \frac{\mathscr{D}f}{\mathscr{D}t}, \tag{15}$$

where $\mathrm{d}f/\mathrm{d}t$ denotes the directional derivative following an acoustic wave, which is the acoustic wave derivative $\mathscr{D}f/\mathscr{D}t$. Combining equations (14) and (15) gives the following relationship between the particle derivative and the wave derivative:

$$\frac{\mathrm{D}f}{\mathrm{D}t} = \frac{\mathscr{D}f}{\mathscr{D}t} - a\mathbf{n} \cdot \nabla f. \tag{16}$$

COMPATIBILITY EQUATIONS

The compatibility equations are linear combinations of the governing equations that consist of directional derivatives in special directions. From physical concepts we know that information propagates through a flowfield by particle motion and acoustic wave motion. Consequently the compatibility equations can be derived by forming the appropriate combinations of the projections of the governing equations in the direction of particle motion and in the direction of acoustic wave motion.

The energy equation, equation (3), contains directional derivatives only in the direction of particle motion. Consequently it is already a compatibility equation applicable along a pathline.

There are no wave derivatives appearing explicitly in the governing equations. Consequently they must be introduced by rearranging the equations. This is accomplished by expressing the particle derivatives in the governing equations in terms of acoustic wave derivatives by using equation (16). The substantial derivatives of velocity $\mathbf{V}$ and pressure P are expressed in terms of acoustic wave derivatives by letting f in equation (16) be $\mathbf{V}$ and P respectively. Those expressions are then substituted into equation (2) and (5) respectively, which are then combined to obtain the waveline compatibility equation.

First let the fluid property f in equation (16) be the particle velocity $\mathbf{V}$. Then

$$\frac{\mathrm{D}\mathbf{V}}{\mathrm{D}t} = \frac{\mathscr{D}\mathbf{V}}{\mathscr{D}t} - a(\mathbf{n} \cdot \nabla)\mathbf{V}. \tag{17}$$

Substituting equation (17) into equation (2) yields

$$\rho\frac{\mathscr{D}\mathbf{V}}{\mathscr{D}t} - \rho a(\mathbf{n}\cdot\nabla)\mathbf{V} + \nabla P = \mathbf{F}. \tag{18}$$

Forming the dot product of $a\mathbf{n}$ with equation (18) gives

$$\rho a\mathbf{n}\cdot\frac{\mathscr{D}\mathbf{V}}{\mathscr{D}t} - \rho a^2\mathbf{n}\cdot(\mathbf{n}\cdot\nabla)\mathbf{V} + a\mathbf{n}\cdot\nabla P = a\mathbf{n}\cdot\mathbf{F}. \tag{19}$$

Next let the fluid property f in equation (16) be the pressure P. Then

$$\frac{\mathrm{D}P}{\mathrm{D}t} = \frac{\mathscr{D}P}{\mathscr{D}t} - a\mathbf{n}\cdot\nabla P. \tag{20}$$

Substituting equation (20) into equation (5) gives

$$\frac{\mathscr{D}P}{\mathscr{D}t} - a\mathbf{n}\cdot\nabla P + \rho a^2\nabla\cdot\mathbf{V} = \psi. \tag{21}$$

Substituting $a\mathbf{n}\cdot\nabla P$ from equation (21) into equation (19) yields

$$\frac{\mathscr{D}P}{\mathscr{D}t} + \rho a\mathbf{n}\cdot\frac{\mathscr{D}\mathbf{V}}{\mathscr{D}t} - \rho a^2[\mathbf{n}\cdot(\mathbf{n}\cdot\nabla)\mathbf{V} - \nabla\cdot\mathbf{V}] = a\mathbf{n}\cdot\mathbf{F} + \psi. \tag{22}$$

Equation (22) is the compatibility equation that is valid on wavelines. It contains directional derivatives of $\mathbf{V}$ and P along the wavelines, some derivatives of $\mathbf{V}$, called cross-derivatives, which are normal to the wavelines, and the source terms $a\mathbf{n}\cdot\mathbf{F} + \psi$. Equation (22) is in vector form and can be applied in any co-ordinate system.

The momentum equation, equation (2), is simply Newton's second law of motion applied to a fluid particle. Since it is a vector equation, it can be written as three independent equations each containing components only in a single spatial direction, e.g. the three co-ordinate directions. Consequently the three component momentum equations are compatibility equations. The three independent directions are arbitrary. The component of the momentum equation, equation (2), in the direction of the arbitrary unit vector $\mathbf{s}$ in physical space is

$$\rho\mathbf{s}\cdot\frac{\mathrm{D}\mathbf{V}}{\mathrm{D}t} + \mathbf{s}\cdot\nabla P = \mathbf{s}\cdot\mathbf{F}. \tag{23}$$

Equation (23) contains derivatives along the pathline in $\mathbf{r}$–t space in the direction of the vector $\mathbf{s}$ in physical (i.e. $\mathbf{r}$) space. Consequently the equation does not apply in one space dimension. Therefore equation (23) is a compatibility equation only in unsteady two- and three-dimensional flows.

In summary, three compatibility equations exist: equations (3), (22) and (23). There are infinite number of wavelines passing through every point in a flowfield corresponding to the infinite number of choices for $\mathbf{n}$, so equation (22) represents an infinite number of waveline compatibility equations. There are two or three independent directions $\mathbf{s}$ in two- or three-dimensional physical space respectively, so equation (23) represents two or three compatibility equations. Consequently there exist an infinite number of compatibility equations at every point in a flowfield.

There are three, four or five governing equations corresponding to one-, two- or three-dimensional flow respectively. Since the compatibility equations are simply linear combinations of the governing equations, there can be only three, four or five independent compatibility equations corresponding to one-, two- or three-dimensional flow respectively. Since the derivative of density appears only in equation (3), the energy equation, the energy equation must be included in any set of independent compatibility equations. Several choices exist for the remaining compatibility equations required to obtain a complete independent set of compatibility equations. The possible complete sets of independent compatibility equations for one-, two- and three-dimensional flow are discussed in the following sections.

UNSTEADY ONE-DIMENSIONAL FLOW

For this case, $\mathbf{r} = \mathbf{i}x$, $\mathbf{V} = \mathbf{i}u$ and $\mathbf{n} = \mathbf{i}n_x$, where $n_x = \pm 1$. Equations (8) and (10) become

$$\mathrm{D}x = u\,\mathrm{D}t \qquad \text{(along pathlines)}, \tag{24}$$

$$\mathscr{D}x_{\pm} = (u \pm a)\mathscr{D}t \qquad \text{(along wavelines)}, \tag{25}$$

where $\mathrm{D}x$ denotes the change in position along the pathline and $\mathscr{D}x_{\pm}$ denotes the change in position along the wavelines. The pathline and the wavelines passing through a solution point are shown in Figure 2, which illustrates the overall physical grid for unsteady one-dimensional flow. Points a, b and c are known initial data points. Point 4 is the solution point and points 1, 2 and 3 are the intersections of the rearward projected characteristics from point 4. The domain of dependence of the differential equations is the region along the initial value line between the outermost paths of information propagation from the initial value domain to the solution point, i.e. the region between the acoustic wavelines, $x_1 \le x \le x_2$. As shown by Courant, *et al.*,[8] for stability the domain of dependence must be included within the convex hull of the difference scheme, which is the physical domain a–b–c.

Equations (3) and (22) become

$$\mathrm{D}P - a^2\mathrm{D}\rho = \psi\,\mathrm{D}t, \tag{26}$$

$$\mathscr{D}P_+ + \rho a\mathscr{D}u_+ = (aX - \delta\rho a^2u/x + \psi)\mathscr{D}t, \tag{27}$$

$$\mathscr{D}P_- - \rho a\mathscr{D}u_- = (-aX - \delta\rho a^2u/x + \psi)\mathscr{D}t, \tag{28}$$

where $\mathrm{D}(\cdot)$ denotes the total differential along the pathline, $\mathscr{D}(\cdot)_+$ and $\mathscr{D}(\cdot)_-$ denote the total differentials along acoustic wavelines corresponding to $n_x = \pm 1$ respectively, X is the

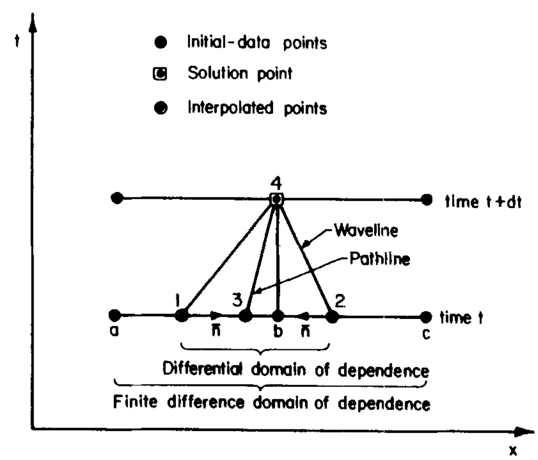

Figure 2. Unsteady one-dimensional flow

x-component of $\mathbf{F}$ and $\delta = 0$, 1 or 2 for Cartesian, cylindrical or spherical co-ordinates respectively. Equations (26)–(28) comprise a complete set of compatibility equations for unsteady one-dimensional flow.

The method of characteristics can be used to identify the conditions that must be satisfied at boundary points. For example, at a solid boundary point, either equation (27) or equation (28) is not applicable, since the waveline along which it applies lies outside of the flowfield. Consequently, only two compatibility equations exist. The third condition required to form a complete set of governing equations is furnished by the condition that $u = 0$ at a solid boundary. Other boundary conditions are implemented in an analogous manner. In all cases the method of characteristics furnishes a clear picture of the physics of the boundary condition.

UNSTEADY TWO-DIMENSIONAL FLOW

For this case, $\mathbf{r} = \mathbf{i}x + \mathbf{j}y$, $\mathbf{V} = \mathbf{i}u + \mathbf{j}v$ and $\mathbf{n} = \mathbf{i}n_x + \mathbf{j}n_y$. Equations (8) and (10) become

$$\mathrm{D}x = u\,\mathrm{D}t, \qquad \mathrm{D}y = v\,\mathrm{D}t, \tag{29}$$

$$\mathscr{D}x = (u + an_x)\mathscr{D}t, \qquad \mathscr{D}y = (v + an_y)\mathscr{D}t. \tag{30}$$

The pathline and the wavelines passing through a point are illustrated in Figure 3.

The domain of dependence of the differential equations is the region in the initial value surface within the outermost paths of propagation from the initial value surface to the solution point. From Figure 3 it is obvious that this is the region within the Mach conoid which is composed of the infinite number of wavelines passing through the solution point. For stability of any numerical method, the differential domain of dependence must lie within the finite difference domain of dependence.

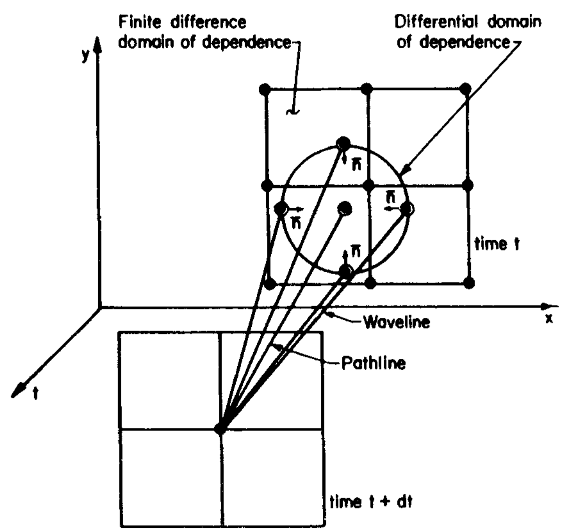

Figure 3. Unsteady two-dimensional flow

Equations (3), (22) and (23) become

$$\mathrm{D}P - a^2\,\mathrm{D}\rho = \psi\,\mathrm{D}t, \tag{31}$$

$$\mathscr{D}P + \rho a(n_x \mathscr{D}u + n_y \mathscr{D}v) - \rho a^2[(n_x^2 - 1)u_x + (n_y^2 - 1)v_x + n_x n_y(u_y + v_x)]\mathscr{D}t$$
$$= [a(n_x X + n_y Y) - \delta\rho a^2 v/y + \psi]\mathscr{D}t, \tag{32}$$

$$\rho s_x\,\mathrm{D}u + \rho s_y\,\mathrm{D}v + (\partial P/\partial s)\mathrm{D}t = (s_x X + s_y Y)\,\mathrm{D}t, \tag{33}$$

where $\delta = 0$ for Cartesian co-ordinates (i.e. planar flow) and $\delta = 1$ for cylindrical co-ordinates (i.e. axisymmetric flow). Equation (32) is valid for any direction **n**. Equation (33) is valid for any independent direction **s**, but for two-dimensional flow, **s** can have only two independent directions. Any other direction will be a linear combination of the first two directions.

There can be only four independent compatibility equations for unsteady two-dimensional flow, and any complete set must include the energy equation, equation (31). The following three complete sets of independent compatibility equations are possible:

(I) equations (31) and (32) for three independent choices for **n**
(II) equations (31) and (32) for two independent choices for **n**, and equation (33) for one value of **s**
(III) equations (31) and (32) for one value of **n**, and equation (33) for two independent choices for **s**.

The choices for the direction of **n** in equation (32) and the direction of **s** in equation (33) are arbitrary, so there are many ways to choose a set of compatibility equations for numerical computations. Considerable simplifications occur in equation (32) when **n** is chosen in the directions of the co-ordinate axes.

Since an infinite number of wavelines are required to define the complete differential domain of dependence and at most three waveline compatibility equations are independent, an exact match between the finite difference domain of dependence and the domain of dependence of the governing equations cannot be obtained.

UNSTEADY THREE-DIMENSIONAL FLOW

For this case, $\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$, $\mathbf{V} = \mathbf{i}u + \mathbf{j}v + \mathbf{k}w$ and $\mathbf{n} = \mathbf{i}n_x + \mathbf{j}n_y + \mathbf{k}n_z$. Equations (8) and (10) become

$$\mathrm{D}x = u\,\mathrm{D}t, \qquad \mathrm{D}y = v\,\mathrm{D}t, \qquad \mathrm{D}z = w\,\mathrm{D}t, \tag{34}$$

$$\mathscr{D}x = (u + an_x)\mathscr{D}t, \qquad \mathscr{D}y = (v + an_y)\mathscr{D}t, \qquad \mathscr{D}z = (w + an_z)\mathscr{D}t. \tag{35}$$

The domain of dependence of the differential equations is the region in the initial value hypersurface within the outermost paths of propagation from the initial value hypersurface to the solution point in $xyzt$ hyperspace. For unsteady three-dimensional flow this region is determined by the intersection of the Mach hyperconoid with the initial value hyperspace. Since the solution hyperspace is four-dimensional (i.e. x, y, z and t), it cannot be illustrated pictorially. The domain of dependence in the initial value space is illustrated in Figure 4. For stability of any numerical method, the differential domain of dependence must lie within the finite difference domain of dependence.

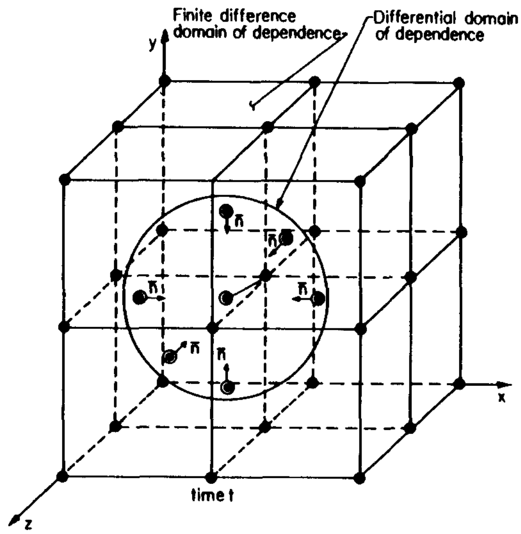

Figure 4. Unsteady three-dimensional flow

Equations (3), (22) and (23) become

$$\mathrm{D}P - a^2\,\mathrm{D}\rho = \psi\,\mathrm{D}t, \tag{36}$$

$$\begin{aligned}\mathscr{D}P + \rho a(n_x\mathscr{D}u + n_y\mathscr{D}v + n_z\mathscr{D}w) - \rho a^2[(n_x^2 - 1)u_x + (n_y^2 - 1)v_y + (n_z^2 - 1)w_z \\ + n_xn_y(u_y + v_x) + n_xn_z(u_z + w_x) + n_yn_z(v_z + w_y)]\mathscr{D}t \\ = [a(n_xX + n_yY + n_zZ) + \psi]\mathscr{D}t,\end{aligned} \tag{37}$$

$$\rho s_x\mathrm{D}u + \rho s_y\mathrm{D}v + \rho s_z\mathrm{D}w + (\partial P/\partial s)\mathrm{D}t = (s_xX + s_yY + s_zZ)\mathrm{D}t. \tag{38}$$

Equation (37) is valid for any direction **n**. Equation (38) is valid for any independent direction **s**, but for three-dimensional flow, **s** can have only three independent directions. Any other direction will be a linear combination of the first three directions.

There can be only five independent compatibility equations for unsteady three-dimensional flow. The following four complete sets of independent compatibility equations are possible:

(I) equations (36) and (37) for four independent choices for **n**
(II) equations (36) and (37) for three independent choices for **n**, and equation (38) for one value of **s**
(III) equations (36) and (37) for two independent choices for **n**, and equation (38) for two independent choices for **s**
(IV) equations (36) and (37) for one value of **n**, and equation (38) for three independent choices for **s**.

The choices for the direction of **n** in equation (37) and the direction of **s** in equation (38) are arbitrary, so there are many ways to choose a set of compatibility equations for numerical computation. Considerable simplifications occur in equation (37) when **n** is chosen in the directions of the co-ordinate axes.

CONCLUSIONS

A simple physical approach for deriving the characteristic and compatibility equations of fluid dynamics has been developed. This approach is based on the physical propagation paths (i.e. pathlines and acoustic wavelines) of information in a flowfield. The characteristic and compatibility equations are derived in vector forms. Complete independent sets of equations for unsteady one-, two- and three-dimensional flows are presented. This simple physical approach to the method of characteristics gives physical insights into the important concepts of propagation paths, domains of dependence and properly posed initial and boundary conditions.

APPENDIX: NOMENCLATURE

a	speed of sound
$\mathbf{F}$	external and viscous forces
$\mathbf{n}$	unit vector in the direction of acoustic wave motion
P	pressure
$\mathbf{r}$	position vector
$\mathbf{s}$	arbitrary unit vector in physical space
t	time
T	temperature
u, v, w	Cartesian velocity components
$\mathbf{V}$	velocity vector
x, y, z	Cartesian co-ordinates
X, Y, Z	x-, y-, z-components of $\mathbf{F}$
ρ	density
ψ	heat transfer, external work and viscous dissipation

Subscripts

x, y, z	pertaining to the x-, y-, z-directions, or partial differentiation with respect to these directions

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